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## Abstract

We study the convergence hypothesis of economic growth relying on recent data. To fix the population size of each country for rank changes, we re-estimate the transition probability matrix by applying a population weight to changes in the income rank of each country. We then find that with no population weighting, the world economy can be divided into two peaks as before. Nevertheless, the population-weighted probability model yields more optimistic results: We divide the world economy into several regions, estimate similar probability models, and calculate the convergence distribution. We then divide the world into the optimistic and pessimistic region. The optimistic region, with high income, is composed of East Asia and Europe; The pessimistic region, with low income, is composed of Sub-Saharan Africa. These two extremes cause the observed twin peaks. The transition information of China and India has a significant impact when considering population weighting. These two countries show rapid economic growth, which produces optimistic results in our population-weighted model.

JEL classification: C49, D39, O50, R11

Keywords: Convergence, World Economy, Markov Chain.

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## 1. Introduction

We study the long-term growth trends of the world economy since the 1950s from the perspective of convergence. We analyze the hypothesis of whether economically poor countries (regions) can catch up with rich countries (regions) in the context of economics. The possibility of convergence increases if we assume a production function with diminishing returns represented à la Solow (1956). This is due to productivity being higher when there are low inputs and gradually decreasing as input increases. We thus link this to empirical research by replacing “low input” with “poor country (region)”.

There is large literature analyzing the trends of convergence at the world scale (Barro and Sala-i-martin, 2004; Islam, 2003). Traditionally, this has been analyzed mainly through econometric methods. In this study, instead, we analyze it relying on the stochastic method proposed by Quah (1993, 1996a, 1996b). This unique analytical method, classified as a “distribution approach” (Islam 2003), produces distinctive results compared to the econometric methodology. It divides the logarithmic value of relative income, which compares each country's GDP per capita with the average, into several ranks in descending order of income. Then, the time-series changes of the rank (income states) in each sample are totaled, and these are represented by probabilities. The Markov transition matrix obtained by such a procedure has an eigenvalue (the dominant eigenvalue), whose absolute value is 1. Therefore, the convergence distribution (ergodic distribution) can be estimated by obtaining the eigenvector associated to the dominant eigenvalue. With this approach, Quah argues that the world economy divides into two extremes with twin peaks: a rich economy and a poor economy.

Up to this point, the changes in the income state have been calculated using a single country (region) as a sample, but it is not easy for the income state to change in countries such as China or India, which have large populations. Taking this into consideration, in this study we multiply each sample by the population weight and we estimate the Markov transition matrix from the aggregated results to obtain the convergence distribution. For the population weight the ratio was obtained from the total of these worlds and was multiplied by the income state movement. This is expected to make the stochastic model more precise, but the aggregation work becomes more complex because the population ratio is multiplied for all states changes.

The rest of this article is organized in the following way. Section 2 describes the stochastic model using Markov chains. Section 3 explains the data used. The world economy is divided into several regions. This aims at finding out the characteristics between regions by conducting similar analyses per region. Section 4 details the results of the regional model without population weights while section 5 does so for the case including population weights. Finally, section 6 concludes.

## 2. The model

The distribution approach of Quah (1993, 1996a, b) relies on a Markov transition matrix to show the change in the distribution from a period to the next. In this section, we briefly explain the essentials of this methodology.

Let  $F_t$  be the income distribution state in period  $t$  expressed as a  $1 \times N$  vector (number of states). The Markov process is a mathematical expression of the situation in which the income distribution situation  $F_{t+1}$  in the next term depends on the income distribution states  $F_t$  in the current term. In other words, the fluctuation of the income distribution states between the two-time points is defined as follows.

$$F_{t+1} = F_t \cdot M_t \quad (1)$$

Where  $M_t$  is the transition matrix. Since the income distribution is ranked for this transition, the transition matrix is estimated by aggregating the changes in the rank. The estimation method is as follows, with  $P$  as the number of changes in rank.

$$M_{t,jk} = P_{t,jk} / \sum_{k=1}^n P_{t,jk} \quad (2)$$

This shows the probability of how many of the total number of changes (total number of income state movements from state  $k$ ), including the number staying in the same state, that moved to state  $j$ . Then, we can take advantage of the ergodic property that is characteristic of Markov chains, and find the convergence distribution (ergodic distribution).

$$F = F \cdot M \quad (3)$$

The transition matrix obtained through this procedure usually has an eigenvalue with an absolute value of 1. Relying on the associated eigenvector, the estimation of the convergence distribution (ergodic distribution) can be obtained using the solver in Excel or through any other method to compute it. In this case, eq (3) holds when  $M$  is given, and the model is constrained so that the sum of  $F$  equal to 1 is solved.

The convergence distribution we analyze is a fixed distribution that is reached when looking at the future, where this model is maintained for a long term in a stochastic model composed of samples from 1953 to 2017. The convergence hypothesis examines whether indicators such as GDP per capita in the world economy, or a specific economic group settle at a certain value. In the verification of the convergence hypothesis by the stochastic model, the shape of the convergence distribution is a concern. Therefore, given that there is convergence to a certain value, it is desirable that the distribution is mountain-shaped, centered on the middle-income group. In the case of other distributions, it cannot be confirmed that the convergence hypothesis is universally established.

### 3. Data

In this study we rely on the Penn World Table (PWT),<sup>3</sup> which provides internationally comparable data (Feenstra, Inklaar and Timmer, 2015). The PWT provides macroeconomic indicators such as GDP, for the case of some countries, presenting estimates going back to 1950. However, since the data for China is only available from 1952 to 2017, we set the period of analysis for that period. We show that the number of countries (regions) change during this period. Figure 1 shows the change in the number of samples in the analysis, with a maximum value of 182. As we can see in the figure, it is possible to classify and analyze the world economy by dividing it into several regions. Table 1 shows the applicable countries when classified by region. Based on this table, we separate it into the following seven regions:

East Asia & Pacific (*ea, es, as*)

Europe & Central Asia (*eu*)

Latin America & Caribbean (*la, nl*)

Middle East & North Africa (*me, ms, as*)

North America (*na, nl*)

South Asia (*sa, es, ms, as*)

Sub-Saharan Africa (*af*)

This regional classification is based on the WDI (World Development Indicators) classification. In addition, for the convenience of the analysis, we combine and analyze the adjacent regions into a larger area (mentioned in parenthesis). For example, (*ea*) in East Asia & Pacific is simply referred to as “East Asia & Pacific”, (*es*) alludes to East Asia & Pacific and South Asia, and (*as*) references to East Asia & Pacific, Middle East & North Africa and South Asia, all combined.

We next use real GDP and population data. The PWT estimates real GDP from production and expenditure, with some slight differences in the estimated values. In this study, we adopt real GDP (“rgdpo” in the PWT notation) from the production side. The relative income per capita is calculated from this data, and is defined as the logarithm when the average income (per year) is 1 ( $\ln(y/n/\sum y/\sum n)$ , with  $y$  being real GDP, and  $n$  the population).

In this paper we analyze the convergence, in fluctuations, of the relative income of each country and region through the lens of the method proposed by Quah (1993, 1996a, 1996b). In this methodology, the logarithmic value of the relative income previously mentioned is ranked in ascending order of income, the change in the rank is followed in chronological order, and the aggregated value is indicated through a probability. Nevertheless, since this ranking is arbitrary, the logarithmic value of the relative income for ranking is fixed with the following numbers based on an average of 0.

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<sup>3</sup> Penn World Table version 9.1, <https://www.rug.nl/ggdc/productivity/pwt/>

5 states: (lo, ml, mi, mh, hi or 1, 2, 3, 4, 5).

-1.386 (1/4), -0.693 (1/2), 0.000 (1), 0.693 (2)

7 states: (ll, lo, ml, mi, mh, hi, hh or 1, 2, 3, 4, 5, 6, 7).

-1.500, -1.000, -0.500, 0.000, 0.500, 1.000

The numbers in parenthesis from the five states show the values for 1/4, 1/2 and 2 times the average income. This allows for changes in the number of samples.

Tables 2 and 4 display the hierarchical distribution by region. In table 2 we see that North America (*na*) only has the highest income group (5) whereas South Asia (*sa*) possesses all but the highest income group. In the case of table 4, *na* only has positive values in states 6 and 7, while *sa* has positive values in all states except in 6 and 7. In regards to the convergence distribution, World (*wd*) is evenly distributed whilst other regions are biased. Among these regions, East Asia & Pacific (*ea*), Europe & Central Asia (*eu*), and Middle East & North Africa (*me*) are concentrated in the high-income group, and Latin America & Caribbean (*la*) is in the middle-income group. Finally, Sub-Saharan Africa (*af*) is concentrated in the low-income group.

Tables 3 and 5 show the changes in the hierarchy of each member of *ea* and *sa*, at a given time. We see that in the seventh state of table 5, Japan falls from a value of 7 to 6. In the case of Australia and Brunei, they remain in the highest income class in every state. In the other extreme, for each state, Nepal persists in the lowest income class. Countries with large populations like China have, in recent years, risen from 2 to 4, while India has lately jumped to 3 after a period of falling from 2 to 1. We would expect that countries with big populations lift their income groups as they grow, although we don't observe this with low-income countries such as Cambodia, Bangladesh, Nepal, and Pakistan.

#### **4. Regional Model of Markov Chain without Population Weight**

We first estimate the stochastic model proposed by Quah (1993, 1996a, 1996b) and find its convergence distribution. The stochastic model estimates models by region as well as the entire world economy and compares their convergence distributions.

Tables 6 and 7 show the transition probability matrices for the world (*wd*) and East Asia & Pacific (*ea*). A five states model (5×5 matrix) and a seven states model (7×7 matrix) are displayed, respectively (see tables 1 and 2 in the Appendix for results of all areas analyzed, including these). Each matrix shows only the elements in which the income state movements are seen. The blank elements are the places where the state movements are not observed, and the movement probability is zero. In addition, since the values are displayed with 4 digits after the decimal point, even if state movements are spotted, they are displayed as 0.0000 if the probability is extremely low. For the convenience of computation, the movement probability is zero at this point as well.

We can observe from table 6 that there are areas where the stochastic model does not hold.

For example, *ea* has a probability of moving from *hi* (highest income group) to *hi* 1.0000. Since there is a probability of moving from *mh* to *hi* in the next income group (0.0561), this shows that once the highest income group is reached, it will not fall to the lower level. Therefore, it is expected that the convergence distribution will be concentrated at one point in *hi*.

Based on these results, the ergodic convergence distribution is calculated in tables 8 and 9. From the above explanation, there are areas where the transition probability matrix does not hold, thus blank parts in the convergence distribution of these areas appear. Additionally, in the table, the income states with the largest distribution are colored yellow, the states with the second largest distribution with blue, and the states with the smallest distribution with orange. Based on this, the tendency of the ergodic convergence distribution in each region is divided as follows. First, only *wd* is divided into two extremes, the highest-income group and the lowest-income group, and the rest is a distribution with a concentration in one peak. Among them, there are many regions that are concentrated in the highest income class, and the regional divisions of *ea*, *eu*, *me*, *na*, and *es* fall under this category. The regional divisions of *la*, *sa*, *nl*, and *ms* are concentrated in the middle-income group, and only *af* (Sub-Saharan Africa) is concentrated in the lowest-income group. This difference in the convergence distribution of each region seems to indicate the “twin peak” that Quah claims, that is, the tendency of the world economy to differentiate into two peaks.

## 5. Regional Model of Markov Chain with Population Weight

Tables 10 and 11 show the transition probability matrix for the world (*wd*) and East Asia & Pacific (*ea*). Since the income state movement itself is irrelevant with or without population weighting, the positions of the elements in which the probabilities are described are the same. By considering the population weight, only the value of the probability is different (see tables 3 and 4 in the Appendix for results for all the regions analyzed, including these). When we compare the values of *wd* in table 6 with those in table 10, the former’s values are 0.9624, 0.9100, 0.9110, 0.9233, 0.9833, whereas the latter’s values are 0.9058, 0.9125, 0.9441, 0.9395, and 0.9976.

Based on this, tables 12 and 13 present the ergodic distributions. There is a clear distinction in each distribution. Initially, the distribution is convergent throughout *wd*, showing a bipolar distribution without population weighting. When we add population weighting, the distribution becomes concentrated in the high-income group. In addition, there are many regions where the distribution is concentrated in the high-income class and, besides *wd*, there are regional divisions corresponding to *ea*, *eu*, *na*, *nl*, *es*, and *as*. The areas concentrated in the middle-income group are divided into two areas, *la* and *me*, while *af* concentrates in the low-income group even when population weighting is taken into consideration. As a result of adding population weight, there is a slight difference in the tendency of the convergence distribution between the five and seven states. Therefore, it becomes difficult to judge the two regional divisions of *sa* and *ms*.

In tables 6 and 10, we compare again this difference in convergence distribution. As we described above, in the elements of the diagonal matrix the probability of *lo* decreases, while the other probabilities increase. Next, the probability of moving from *lo* to *ml* increases from 0.0370 without population weighting to 0.0942 with population weighting. From this, we can say that the population weighted model is more likely to get out of the low-income class. Furthermore, the probability that the income class will rise from *ml* to *mi*, *mi* to *mh*, and *mh* to *hi* is higher without population weight, whereas the probability that the income class will fall from *hi* to *mh*, *mh* to *mi*, and *mi* to *ml* is higher with population weight. It is likely that the distribution will be concentrated in the high-income group, since no alternative is higher. So, we can consider that the discrepancy in the result of the convergence distribution is due to such a difference in probability.

## 6. Conclusion

We analyzed the convergence distribution of the world economy by region using a stochastic model based on a Quah-style Markov chain. As a feature of the study, we compared a normal stochastic model without population weighting with a model that considers it. We found that the model without population weighting still shows the tendency of the world economy to become polarized (Twin Peaks). One of the factors for this is the characteristics of each region. Sub-Saharan Africa has a convergent distribution of low incomes, but many regions have high or medium incomes.

On the other hand, when we considered population weighting, the distribution tended to be concentrated in the high-income group, indicating that the possibility of twin peaks is excluded. However, it is still regional and sub-Saharan Africa continues being a low-income region. The convergence hypothesis does not hold without population weighting, but it does so with population weighting. Nevertheless, the fact that income is concentrated in high-income groups does not suggest that income inequality will disappear in the foreseeable future.

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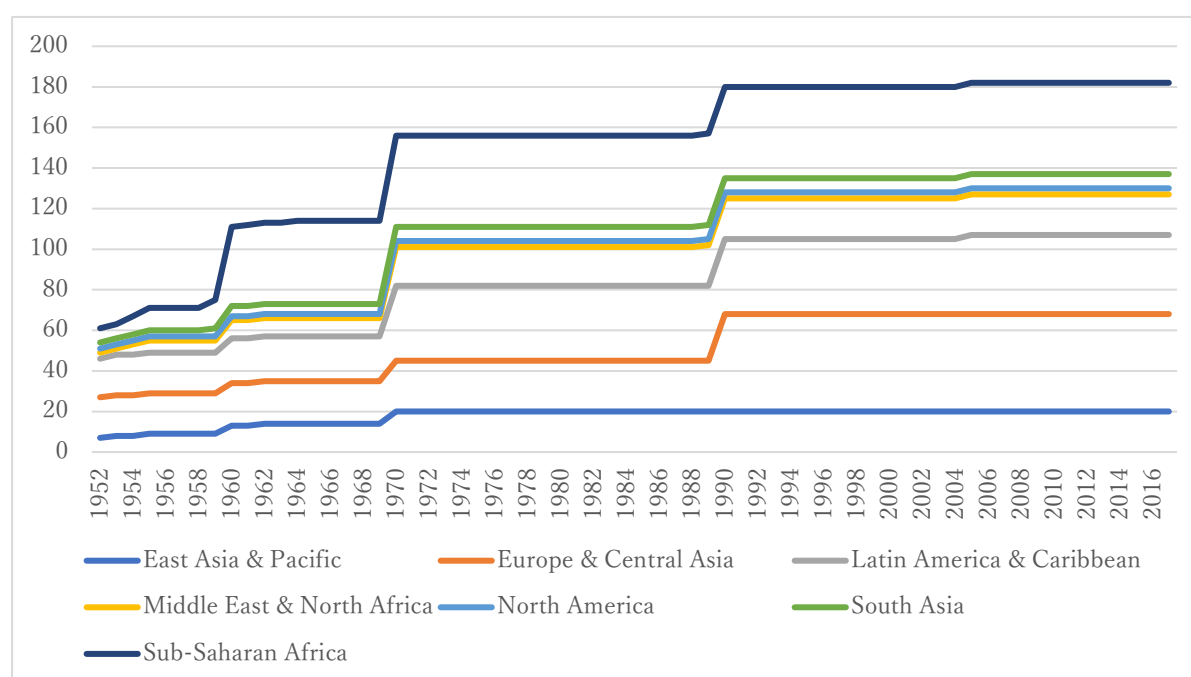
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Table 1 Classification of Region

|                      |                            |               |            |
|----------------------|----------------------------|---------------|------------|
| East Asia & Pacific  | Middle East & North Africa | North America | South Asia |
| Australia            | United Arab Emirates       | Bermuda       | Bangladesh |
| Brunei Darussalam    | Bahrain                    | Canada        | Bhutan     |
| China                | Djibouti                   | United States | India      |
| Fiji                 | Algeria                    |               | Sri Lanka  |
| China, Hong Kong SAR | Egypt                      |               | Maldives   |
| Indonesia            | Iran (Islamic Republic of) |               | Nepal      |
| Japan                | Iraq                       |               | Pakistan   |
| Cambodia             | Israel                     |               |            |
| Republic of Korea    | Jordan                     |               |            |
| Lao People's DR      | Kuwait                     |               |            |
| China, Macao SAR     | Lebanon                    |               |            |
| Myanmar              | Morocco                    |               |            |
| Mongolia             | Malta                      |               |            |
| Malaysia             | Oman                       |               |            |
| New Zealand          | State of Palestine         |               |            |
| Philippines          | Qatar                      |               |            |
| Singapore            | Saudi Arabia               |               |            |
| Thailand             | Syrian Arab Republic       |               |            |
| Taiwan               | Tunisia                    |               |            |
| Viet Nam             | Yemen                      |               |            |

Figure 1 Number of Countries



## Continues

| Europe & Central Asia  |                     | Latin America & Caribbean        |                                    | Sub-Saharan Africa       |                            |
|------------------------|---------------------|----------------------------------|------------------------------------|--------------------------|----------------------------|
| Albania                | Italy               | Aruba                            | Mexico                             | Angola                   | Mozambique                 |
| Armenia                | Kazakhstan          | Anguilla                         | Montserrat                         | Burundi                  | Mauritania                 |
| Austria                | Kyrgyzstan          | Argentina                        | Nicaragua                          | Benin                    | Mauritius                  |
| Azerbaijan             | Lithuania           | Antigua and Barbuda              | Panama                             | Burkina Faso             | Malawi                     |
| Belgium                | Luxembourg          | Bahamas                          | Peru                               | Botswana                 | Namibia                    |
| Bulgaria               | Latvia              | Belize                           | Paraguay                           | Central African Republic | Niger                      |
| Bosnia and Herzegovina | Republic of Moldova | Bolivia (Plurinational State of) | El Salvador                        | Côte d'Ivoire            | Nigeria                    |
| Belarus                | North Macedonia     | Brazil                           | Suriname                           | Cameroon                 | Rwanda                     |
| Switzerland            | Montenegro          | Barbados                         | Sint Maarten (Dutch part)          | D.R. of the Congo        | Sudan                      |
| Cyprus                 | Netherlands         | Chile                            | Turks and Caicos Islands           | Congo                    | Senegal                    |
| Czech Republic         | Norway              | Colombia                         | Trinidad and Tobago                | Comoros                  | Sierra Leone               |
| Germany                | Poland              | Costa Rica                       | Uruguay                            | Cabo Verde               | Sao Tome and Principe      |
| Denmark                | Portugal            | Curaçao                          | St. Vincent and the Grenadines     | Ethiopia                 | Eswatini                   |
| Spain                  | Romania             | Cayman Islands                   | Venezuela (Bolivarian Republic of) | Gabon                    | Seychelles                 |
| Estonia                | Russian Federation  | Dominica                         | British Virgin Islands             | Ghana                    | Chad                       |
| Finland                | Serbia              | Dominican Republic               |                                    | Guinea                   | Togo                       |
| France                 | Slovakia            | Ecuador                          |                                    | Gambia                   | U.R. of Tanzania: Mainland |
| United Kingdom         | Slovenia            | Grenada                          |                                    | Guinea-Bissau            | Uganda                     |
| Georgia                | Sweden              | Guatemala                        |                                    | Equatorial Guinea        | South Africa               |
| Greece                 | Tajikistan          | Honduras                         |                                    | Kenya                    | Zambia                     |
| Croatia                | Turkmenistan        | Haiti                            |                                    | Liberia                  | Zimbabwe                   |
| Hungary                | Turkey              | Jamaica                          |                                    | Lesotho                  |                            |
| Ireland                | Ukraine             | Saint Kitts and Nevis            |                                    | Madagascar               |                            |
| Iceland                | Uzbekistan          | Saint Lucia                      |                                    | Mali                     |                            |

Table 2 Distribution of Income State among Region (Five States)

|                            | 1     | 2     | 3     | 4     | 5     |
|----------------------------|-------|-------|-------|-------|-------|
| All wd                     | 1,820 | 1,869 | 2,088 | 1,774 | 2,319 |
| East Asia & Pacific        | 195   | 248   | 209   | 109   | 405   |
| Europe & Central Asia      | 43    | 118   | 314   | 647   | 1,092 |
| Latin America & Caribbean  | 50    | 320   | 881   | 631   | 299   |
| Middle East & North Africa | 91    | 223   | 268   | 183   | 307   |
| North America              | 0     | 0     | 0     | 0     | 180   |
| South Asia                 | 151   | 172   | 77    | 11    | 0     |
| Sub-Saharan Africa         | 1,290 | 788   | 339   | 193   | 36    |

Table 3 Income State Changes (Five States, East Asia & Pacific, South Asia)

|                      | 1952 | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 | 2017 |
|----------------------|------|------|------|------|------|------|------|------|
| Australia            | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    |
| Brunei Darussalam    |      |      | 5    | 5    | 5    | 5    | 5    | 5    |
| China                | 2    | 2    | 2    | 2    | 2    | 2    | 3    | 3    |
| Fiji                 |      | 3    | 3    | 3    | 3    | 3    | 3    | 3    |
| China, Hong Kong SAR |      | 4    | 4    | 5    | 5    | 5    | 5    | 5    |
| Indonesia            |      | 2    | 1    | 2    | 2    | 2    | 3    | 3    |
| Japan                | 3    | 4    | 5    | 5    | 5    | 5    | 5    | 5    |
| Cambodia             |      |      | 2    | 1    | 1    | 1    | 1    | 1    |
| Republic of Korea    |      | 2    | 2    | 3    | 4    | 5    | 5    | 5    |
| Lao People's DR      |      |      | 1    | 1    | 1    | 1    | 2    | 2    |
| China, Macao SAR     |      |      | 4    | 5    | 5    | 5    | 5    | 5    |
| Myanmar              |      |      | 1    | 1    | 1    | 1    | 2    | 2    |
| Mongolia             |      |      | 1    | 2    | 2    | 2    | 3    | 3    |
| Malaysia             |      | 3    | 3    | 3    | 4    | 4    | 4    | 4    |
| New Zealand          | 5    | 5    | 5    | 5    | 5    | 5    | 5    | 5    |
| Philippines          | 2    | 2    | 2    | 2    | 2    | 2    | 2    | 3    |
| Singapore            |      | 3    | 4    | 5    | 5    | 5    | 5    | 5    |
| Thailand             | 2    | 2    | 2    | 2    | 3    | 3    | 3    | 4    |
| Taiwan               | 2    | 3    | 3    | 4    | 5    | 5    | 5    | 5    |
| Viet Nam             |      |      | 1    | 1    | 1    | 1    | 2    | 2    |
| Bangladesh           |      | 2    | 2    | 1    | 1    | 1    | 1    | 1    |
| Bhutan               |      |      | 2    | 2    | 2    | 2    | 3    | 3    |
| India                | 1    | 2    | 1    | 1    | 1    | 1    | 2    | 2    |
| Sri Lanka            | 3    | 3    | 3    | 2    | 2    | 2    | 3    | 3    |
| Maldives             |      |      | 2    | 3    | 3    | 3    | 3    | 3    |
| Nepal                |      | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| Pakistan             | 2    | 2    | 2    | 2    | 2    | 2    | 2    | 2    |

Table 4 Distribution of Income State among Region (Seven States)

|                            | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|----------------------------|-------|-------|-------|-------|-------|-------|-------|
| All wd                     | 1,584 | 1,230 | 1,459 | 1,504 | 1,427 | 1,058 | 1,608 |
| East Asia & Pacific        | 163   | 154   | 189   | 146   | 90    | 117   | 307   |
| Europe & Central Asia      | 28    | 69    | 126   | 252   | 507   | 503   | 729   |
| Latin America & Caribbean  | 39    | 110   | 447   | 655   | 528   | 227   | 175   |
| Middle East & North Africa | 78    | 104   | 214   | 186   | 128   | 145   | 217   |
| North America              | 0     | 0     | 0     | 0     | 0     | 5     | 175   |
| South Asia                 | 129   | 138   | 90    | 43    | 11    | 0     | 0     |
| Sub-Saharan Africa         | 1,147 | 655   | 393   | 222   | 163   | 61    | 5     |

Table 5 Income State Changes (Seven States, East Asia & Pacific, South Asia)

|                      | 1952 | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 | 2017 |
|----------------------|------|------|------|------|------|------|------|------|
| Australia            | 7    | 7    | 7    | 7    | 7    | 7    | 7    | 7    |
| Brunei Darussalam    |      |      | 7    | 7    | 7    | 7    | 7    | 7    |
| China                | 2    | 2    | 2    | 2    | 2    | 3    | 4    | 4    |
| Fiji                 |      | 4    | 4    | 4    | 4    | 4    | 3    | 3    |
| China, Hong Kong SAR |      | 5    | 5    | 6    | 7    | 7    | 7    | 6    |
| Indonesia            |      | 2    | 2    | 2    | 3    | 3    | 3    | 4    |
| Japan                | 4    | 5    | 6    | 7    | 7    | 7    | 7    | 6    |
| Cambodia             |      |      | 2    | 1    | 1    | 1    | 1    | 1    |
| Republic of Korea    |      | 2    | 3    | 4    | 5    | 6    | 6    | 6    |
| Lao People's DR      |      |      | 1    | 1    | 1    | 1    | 2    | 3    |
| China, Macao SAR     |      |      | 5    | 6    | 7    | 7    | 7    | 7    |
| Myanmar              |      |      | 1    | 1    | 1    | 1    | 2    | 3    |
| Mongolia             |      |      | 2    | 2    | 3    | 2    | 3    | 4    |
| Malaysia             |      | 3    | 4    | 4    | 5    | 5    | 5    | 5    |
| New Zealand          | 7    | 7    | 7    | 6    | 6    | 7    | 6    | 6    |
| Philippines          | 3    | 3    | 3    | 3    | 3    | 3    | 3    | 3    |
| Singapore            |      | 4    | 5    | 7    | 7    | 7    | 7    | 7    |
| Thailand             | 2    | 2    | 3    | 3    | 4    | 4    | 4    | 5    |
| Taiwan               | 3    | 3    | 4    | 5    | 6    | 7    | 7    | 7    |
| Viet Nam             |      |      | 1    | 1    | 1    | 2    | 2    | 3    |
| Bangladesh           |      | 2    | 2    | 1    | 1    | 1    | 1    | 2    |
| Bhutan               |      |      | 2    | 2    | 2    | 3    | 3    | 3    |
| India                | 2    | 2    | 2    | 1    | 1    | 1    | 2    | 3    |
| Sri Lanka            | 4    | 4    | 3    | 2    | 3    | 3    | 4    | 4    |
| Maldives             |      |      | 3    | 3    | 4    | 4    | 4    | 4    |
| Nepal                |      | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| Pakistan             | 2    | 2    | 2    | 2    | 2    | 2    | 2    | 2    |

Table 6 Five States Markov Transition Matrix without Population Weight (*wd* and *ea*)

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| wd | lo | 0.9624 | 0.0370 |        | 0.0006 |        |
| wd | ml | 0.0428 | 0.9100 | 0.0472 |        |        |
| wd | mi | 0.0015 | 0.0401 | 0.9110 | 0.0474 |        |
| wd | mh |        | 0.0006 | 0.0502 | 0.9233 | 0.0260 |
| wd | hi | 0.0004 |        | 0.0004 | 0.0158 | 0.9833 |
|    |    |        |        |        |        |        |
| ea | lo | 0.9330 | 0.0670 |        |        |        |
| ea | ml | 0.0408 | 0.9102 | 0.0490 |        |        |
| ea | mi |        | 0.0245 | 0.9314 | 0.0441 |        |
| ea | mh |        |        | 0.0280 | 0.9159 | 0.0561 |
| ea | hi |        |        |        |        | 1.0000 |

Table 7 Seven States Markov Transition Matrix without Population Weight (*wd* and *ea*)

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| wd | ll | 0.9620 | 0.0368 | 0.0006 |        | 0.0006 |        |        |
| wd | lo | 0.0584 | 0.8774 | 0.0626 | 0.0016 |        |        |        |
| wd | ml | 0.0007 | 0.0523 | 0.8885 | 0.0578 |        | 0.0007 |        |
| wd | mi | 0.0014 | 0.0007 | 0.0543 | 0.8784 | 0.0645 | 0.0007 |        |
| wd | mh |        | 0.0007 | 0.0007 | 0.0608 | 0.8933 | 0.0437 | 0.0007 |
| wd | hi |        |        | 0.0010 | 0.0010 | 0.0505 | 0.8873 | 0.0603 |
| wd | hh | 0.0006 |        |        |        |        | 0.0378 | 0.9616 |
|    |    |        |        |        |        |        |        |        |
| ea | ll | 0.9383 | 0.0617 |        |        |        |        |        |
| ea | lo | 0.0519 | 0.8701 | 0.0779 |        |        |        |        |
| ea | ml |        | 0.0217 | 0.9348 | 0.0435 |        |        |        |
| ea | mi |        |        | 0.0140 | 0.9231 | 0.0629 |        |        |
| ea | mh |        |        |        | 0.0341 | 0.8977 | 0.0682 |        |
| ea | hi |        |        |        |        |        | 0.9027 | 0.0973 |
| ea | hh |        |        |        |        |        | 0.0298 | 0.9702 |

Table 8 Five States Ergodic Distribution without Population Weight

|    | lo     | ml     | mi     | mh     | hi     |
|----|--------|--------|--------|--------|--------|
| wd | 0.2015 | 0.1678 | 0.1879 | 0.1731 | 0.2697 |
| ea |        |        |        |        | 1.0000 |
| eu | 0.0113 | 0.0383 | 0.1003 | 0.2383 | 0.6118 |
| la | 0.0367 | 0.1773 | 0.4022 | 0.2882 | 0.0957 |
| me | 0.0611 | 0.1744 | 0.2382 | 0.1824 | 0.3439 |
| na | 1.0000 |        |        |        |        |
| sa | 0.3446 | 0.3932 | 0.2282 | 0.0339 |        |
| af | 0.6335 | 0.2340 | 0.0800 | 0.0433 | 0.0092 |
| nl | 0.0347 | 0.1678 | 0.3805 | 0.2727 | 0.1443 |
| es |        |        |        |        | 1.0000 |
| ms | 0.1246 | 0.2290 | 0.2375 | 0.1473 | 0.2616 |
| as | 0.0685 | 0.1192 | 0.1462 | 0.1235 | 0.5426 |

Table 9 Seven States Ergodic Distribution without Population Weight

|    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|--------|--------|--------|--------|--------|--------|--------|
| wd | 0.1820 | 0.1118 | 0.1307 | 0.1334 | 0.1375 | 0.1175 | 0.1870 |
| ea |        |        |        |        |        | 0.2344 | 0.7656 |
| eu | 0.0058 | 0.0176 | 0.0356 | 0.0738 | 0.1735 | 0.2255 | 0.4682 |
| la | 0.0340 | 0.0699 | 0.2270 | 0.2954 | 0.2420 | 0.0797 | 0.0519 |
| me | 0.0437 | 0.0732 | 0.1714 | 0.1749 | 0.1344 | 0.1922 | 0.2102 |
| na |        |        |        |        |        | 0.0335 | 0.9665 |
| sa | 0.2367 | 0.2515 | 0.2713 | 0.1896 | 0.0509 |        |        |
| af | 0.6061 | 0.2166 | 0.0973 | 0.0392 | 0.0275 | 0.0122 | 0.0010 |
| nl | 0.0323 | 0.0665 | 0.2162 | 0.2814 | 0.2306 | 0.0774 | 0.0956 |
| es |        |        |        |        |        | 0.2344 | 0.7656 |
| ms | 0.0901 | 0.1211 | 0.1887 | 0.1712 | 0.1143 | 0.1502 | 0.1643 |
| as | 0.0365 | 0.0447 | 0.0873 | 0.0979 | 0.0920 | 0.2217 | 0.4199 |

Table 10 Five States Markov Transition Matrix with Population Weight (*wd* and *ea*)

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| wd | lo | 0.9058 | 0.0942 |        | 0.0000 |        |
| wd | ml | 0.0632 | 0.9125 | 0.0243 |        |        |
| wd | mi | 0.0022 | 0.0195 | 0.9441 | 0.0342 |        |
| wd | mh |        | 0.0000 | 0.0398 | 0.9395 | 0.0207 |
| wd | hi | 0.0000 |        | 0.0000 | 0.0024 | 0.9976 |
|    |    |        |        |        |        |        |
| ea | lo | 0.7134 | 0.2866 |        |        |        |
| ea | ml | 0.0636 | 0.9106 | 0.0258 |        |        |
| ea | mi |        | 0.0136 | 0.9653 | 0.0210 |        |
| ea | mh |        |        | 0.0330 | 0.9006 | 0.0664 |
| ea | hi |        |        |        |        | 1.0000 |

Table 11 Seven States Markov Transition Matrix with Population Weight (*wd* and *ea*)

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| wd | ll | 0.9231 | 0.0769 | 0.0000 |        | 0.0000 |        |        |
| wd | lo | 0.0385 | 0.9153 | 0.0462 | 0.0000 |        |        |        |
| wd | ml | 0.0000 | 0.0484 | 0.9046 | 0.0470 |        | 0.0000 |        |
| wd | mi | 0.0032 | 0.0001 | 0.0213 | 0.9263 | 0.0492 | 0.0000 |        |
| wd | mh |        | 0.0000 | 0.0002 | 0.0481 | 0.9041 | 0.0475 | 0.0001 |
| wd | hi |        |        | 0.0000 | 0.0000 | 0.0403 | 0.9049 | 0.0548 |
| wd | hh | 0.0000 |        |        |        |        | 0.0164 | 0.9836 |
|    |    |        |        |        |        |        |        |        |
| ea | ll | 0.6988 | 0.3012 |        |        |        |        |        |
| ea | lo | 0.0323 | 0.9111 | 0.0566 |        |        |        |        |
| ea | ml |        | 0.0505 | 0.9066 | 0.0430 |        |        |        |
| ea | mi |        |        | 0.0001 | 0.9633 | 0.0366 |        |        |
| ea | mh |        |        |        | 0.0409 | 0.8747 | 0.0844 |        |
| ea | hi |        |        |        |        |        | 0.9188 | 0.0812 |
| ea | hh |        |        |        |        |        | 0.0460 | 0.9540 |



Table 12 Five States Ergodic Distribution with Population Weight

|    | lo     | ml     | mi     | mh     | hi     |
|----|--------|--------|--------|--------|--------|
| wd | 0.0572 | 0.0820 | 0.0920 | 0.0791 | 0.6896 |
| ea |        |        |        |        | 1.0000 |
| eu | 0.0009 | 0.0058 | 0.0227 | 0.1006 | 0.8700 |
| la | 0.0093 | 0.0542 | 0.5453 | 0.3877 | 0.0036 |
| me | 0.0901 | 0.1812 | 0.3751 | 0.2647 | 0.0889 |
| na | 1.0000 |        |        |        |        |
| sa | 0.5605 | 0.4281 | 0.0114 | 0.0000 |        |
| af | 0.6865 | 0.2214 | 0.0666 | 0.0254 | 0.0001 |
| nl | 0.0048 | 0.0279 | 0.2806 | 0.1995 | 0.4872 |
| es |        |        |        |        | 1.0000 |
| ms | 0.4497 | 0.3713 | 0.0982 | 0.0605 | 0.0203 |
| as | 0.0620 | 0.1054 | 0.1154 | 0.0658 | 0.6514 |

Table 13 Seven States Ergodic Distribution with Population Weight

|    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|--------|--------|--------|--------|--------|--------|--------|
| wd | 0.0404 | 0.0711 | 0.0602 | 0.1144 | 0.1166 | 0.1378 | 0.4595 |
| ea |        |        |        |        |        | 0.3613 | 0.6387 |
| eu | 0.0006 | 0.0019 | 0.0144 | 0.0320 | 0.1297 | 0.2607 | 0.5608 |
| la | 0.0181 | 0.0241 | 0.1436 | 0.4336 | 0.3402 | 0.0395 | 0.0009 |
| me | 0.0566 | 0.0634 | 0.2057 | 0.2885 | 0.2783 | 0.0668 | 0.0409 |
| na |        |        |        |        |        | 0.0000 | 1.0000 |
| sa | 0.1142 | 0.1607 | 0.6726 | 0.0521 | 0.0004 |        |        |
| af | 0.6868 | 0.1998 | 0.0780 | 0.0248 | 0.0106 | 0.0001 | 0.0000 |
| nl | 0.0107 | 0.0143 | 0.0852 | 0.2575 | 0.2020 | 0.0235 | 0.4067 |
| es |        |        |        |        |        | 0.3613 | 0.6387 |
| ms | 0.1639 | 0.2132 | 0.2360 | 0.1752 | 0.1526 | 0.0366 | 0.0224 |
| as | 0.0092 | 0.0228 | 0.0206 | 0.0767 | 0.0631 | 0.3227 | 0.4849 |

Appendix Table 1 Five States Markov Transition Matrix without Population Weight

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| wd | lo | 0.9624 | 0.0370 |        | 0.0006 |        |
| wd | ml | 0.0428 | 0.9100 | 0.0472 |        |        |
| wd | mi | 0.0015 | 0.0401 | 0.9110 | 0.0474 |        |
| wd | mh |        | 0.0006 | 0.0502 | 0.9233 | 0.0260 |
| wd | hi | 0.0004 |        | 0.0004 | 0.0158 | 0.9833 |
| ea | lo | 0.9330 | 0.0670 |        |        |        |
| ea | ml | 0.0408 | 0.9102 | 0.0490 |        |        |
| ea | mi |        | 0.0245 | 0.9314 | 0.0441 |        |
| ea | mh |        |        | 0.0280 | 0.9159 | 0.0561 |
| ea | hi |        |        |        |        | 1.0000 |
| eu | lo | 0.8837 | 0.1163 |        |        |        |
| eu | ml | 0.0345 | 0.8534 | 0.1121 |        |        |
| eu | mi |        | 0.0428 | 0.8816 | 0.0757 |        |
| eu | mh |        |        | 0.0318 | 0.9490 | 0.0191 |
| eu | hi |        |        |        | 0.0074 | 0.9926 |
| la | lo | 0.9167 | 0.0625 |        | 0.0208 |        |
| la | ml | 0.0128 | 0.9295 | 0.0577 |        |        |
| la | mi | 0.0012 | 0.0242 | 0.9308 | 0.0438 |        |
| la | mh |        | 0.0016 | 0.0600 | 0.9206 | 0.0178 |
| la | hi | 0.0034 |        | 0.0034 | 0.0470 | 0.9463 |
| me | lo | 0.8556 | 0.1444 |        |        |        |
| me | ml | 0.0455 | 0.8818 | 0.0727 |        |        |
| me | mi | 0.0038 | 0.0494 | 0.9125 | 0.0342 |        |
| me | mh |        |        | 0.0447 | 0.9050 | 0.0503 |
| me | hi |        |        |        | 0.0267 | 0.9733 |
| na | lo | 1.0000 |        |        |        |        |
| na | ml |        | 1.0000 |        |        |        |
| na | mi |        |        | 1.0000 |        |        |
| na | mh |        |        |        | 1.0000 |        |
| na | hi |        |        |        |        | 1.0000 |

Continues

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| sa | lo | 0.9799 | 0.0201 |        |        |        |
| sa | ml | 0.0176 | 0.9353 | 0.0471 |        |        |
| sa | mi |        | 0.0811 | 0.8919 | 0.0270 |        |
| sa | mh |        |        | 0.1818 | 0.8182 |        |
| sa | hi |        |        |        |        | 1.0000 |
| af | lo | 0.9769 | 0.0231 |        |        |        |
| af | ml | 0.0614 | 0.9130 | 0.0256 |        |        |
| af | mi | 0.0030 | 0.0719 | 0.8772 | 0.0479 |        |
| af | mh |        |        | 0.0885 | 0.8750 | 0.0365 |
| af | hi |        |        |        | 0.1714 | 0.8286 |
| nl | lo | 0.9167 | 0.0625 |        | 0.0208 |        |
| nl | ml | 0.0128 | 0.9295 | 0.0577 |        |        |
| nl | mi | 0.0012 | 0.0242 | 0.9308 | 0.0438 |        |
| nl | mh |        | 0.0016 | 0.0600 | 0.9206 | 0.0178 |
| nl | hi | 0.0021 |        | 0.0021 | 0.0295 | 0.9663 |
| es | lo | 0.9534 | 0.0466 |        |        |        |
| es | ml | 0.0313 | 0.9205 | 0.0482 |        |        |
| es | mi |        | 0.0396 | 0.9209 | 0.0396 |        |
| es | mh |        |        | 0.0424 | 0.9068 | 0.0508 |
| es | hi |        |        |        |        | 1.0000 |
| ms | lo | 0.9331 | 0.0669 |        |        |        |
| ms | ml | 0.0333 | 0.9051 | 0.0615 |        |        |
| ms | mi | 0.0030 | 0.0564 | 0.9080 | 0.0326 |        |
| ms | mh |        |        | 0.0526 | 0.9000 | 0.0474 |
| ms | hi |        |        |        | 0.0267 | 0.9733 |
| as | lo | 0.9330 | 0.0670 |        |        |        |
| as | ml | 0.0362 | 0.9071 | 0.0567 |        |        |
| as | mi | 0.0018 | 0.0444 | 0.9168 | 0.0370 |        |
| as | mh |        |        | 0.0438 | 0.9057 | 0.0505 |
| as | hi |        |        |        | 0.0115 | 0.9885 |

Appendix Table 2 Seven States Markov Transition Matrix without Population Weight

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| wd | ll | 0.9620 | 0.0368 | 0.0006 |        | 0.0006 |        |        |
| wd | lo | 0.0584 | 0.8774 | 0.0626 | 0.0016 |        |        |        |
| wd | ml | 0.0007 | 0.0523 | 0.8885 | 0.0578 |        | 0.0007 |        |
| wd | mi | 0.0014 | 0.0007 | 0.0543 | 0.8784 | 0.0645 | 0.0007 |        |
| wd | mh |        | 0.0007 | 0.0007 | 0.0608 | 0.8933 | 0.0437 | 0.0007 |
| wd | hi |        |        | 0.0010 | 0.0010 | 0.0505 | 0.8873 | 0.0603 |
| wd | hh | 0.0006 |        |        |        |        | 0.0378 | 0.9616 |
| ea | ll | 0.9383 | 0.0617 |        |        |        |        |        |
| ea | lo | 0.0519 | 0.8701 | 0.0779 |        |        |        |        |
| ea | ml |        | 0.0217 | 0.9348 | 0.0435 |        |        |        |
| ea | mi |        |        | 0.0140 | 0.9231 | 0.0629 |        |        |
| ea | mh |        |        |        | 0.0341 | 0.8977 | 0.0682 |        |
| ea | hi |        |        |        |        |        | 0.9027 | 0.0973 |
| ea | hh |        |        |        |        |        | 0.0298 | 0.9702 |
| eu | ll | 0.8214 | 0.1786 |        |        |        |        |        |
| eu | lo | 0.0588 | 0.7941 | 0.1471 |        |        |        |        |
| eu | ml |        | 0.0640 | 0.8320 | 0.1040 |        |        |        |
| eu | mi |        | 0.0041 | 0.0413 | 0.8595 | 0.0950 |        |        |
| eu | mh |        |        | 0.0020 | 0.0384 | 0.9091 | 0.0505 |        |
| eu | hi |        |        |        |        | 0.0389 | 0.9121 | 0.0491 |
| eu | hh |        |        |        |        |        | 0.0236 | 0.9764 |
| la | ll | 0.9189 | 0.0541 |        |        | 0.0270 |        |        |
| la | lo | 0.0278 | 0.9074 | 0.0556 | 0.0093 |        |        |        |
| la | ml | 0.0023 | 0.0183 | 0.9128 | 0.0642 |        | 0.0023 |        |
| la | mi |        |        | 0.0526 | 0.8901 | 0.0557 | 0.0015 |        |
| la | mh |        | 0.0019 |        | 0.0698 | 0.8973 | 0.0310 |        |
| la | hi |        |        | 0.0045 | 0.0045 | 0.0938 | 0.8304 | 0.0670 |
| la | hh | 0.0057 |        |        |        |        | 0.0971 | 0.8971 |
| me | ll | 0.8961 | 0.0909 | 0.0130 |        |        |        |        |
| me | lo | 0.0490 | 0.8235 | 0.1275 |        |        |        |        |
| me | ml |        | 0.0521 | 0.8531 | 0.0948 |        |        |        |
| me | mi | 0.0055 |        | 0.0874 | 0.8579 | 0.0492 |        |        |
| me | mh |        |        |        | 0.0640 | 0.8640 | 0.0640 | 0.0080 |
| me | hi |        |        |        |        | 0.0504 | 0.8993 | 0.0504 |
| me | hh |        |        |        |        |        | 0.0512 | 0.9488 |
| na | ll | 1.0000 |        |        |        |        |        |        |
| na | lo |        | 1.0000 |        |        |        |        |        |
| na | ml |        |        | 1.0000 |        |        |        |        |
| na | mi |        |        |        | 1.0000 |        |        |        |
| na | mh |        |        |        |        | 1.0000 |        |        |
| na | hi |        |        |        |        |        | 0.5000 | 0.5000 |
| na | hh |        |        |        |        |        | 0.0173 | 0.9827 |

Continues

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| sa | ll | 0.9766 | 0.0234 |        |        |        |        |        |
| sa | lo | 0.0221 | 0.9412 | 0.0368 |        |        |        |        |
| sa | ml |        | 0.0341 | 0.9318 | 0.0341 |        |        |        |
| sa | mi |        |        | 0.0488 | 0.9024 | 0.0488 |        |        |
| sa | mh |        |        |        | 0.1818 | 0.8182 |        |        |
| sa | hi |        |        |        |        |        | 1.0000 |        |
| sa | hh |        |        |        |        |        |        | 1.0000 |
| af | ll | 0.9732 | 0.0268 |        |        |        |        |        |
| af | lo | 0.0742 | 0.8779 | 0.0464 | 0.0015 |        |        |        |
| af | ml |        | 0.1049 | 0.8670 | 0.0281 |        |        |        |
| af | mi | 0.0046 |        | 0.0737 | 0.8479 | 0.0737 |        |        |
| af | mh |        |        |        | 0.1049 | 0.8580 | 0.0370 |        |
| af | hi |        |        |        |        | 0.0833 | 0.8667 | 0.0500 |
| af | hh |        |        |        |        |        | 0.6000 | 0.4000 |
| nl | ll | 0.9189 | 0.0541 |        |        | 0.0270 |        |        |
| nl | lo | 0.0278 | 0.9074 | 0.0556 | 0.0093 |        |        |        |
| nl | ml | 0.0023 | 0.0183 | 0.9128 | 0.0642 |        | 0.0023 |        |
| nl | mi |        |        | 0.0526 | 0.8901 | 0.0557 | 0.0015 |        |
| nl | mh |        | 0.0019 |        | 0.0698 | 0.8973 | 0.0310 |        |
| nl | hi |        |        | 0.0044 | 0.0044 | 0.0921 | 0.8246 | 0.0746 |
| nl | hh | 0.0029 |        |        |        |        | 0.0575 | 0.9397 |
| es | ll | 0.9552 | 0.0448 |        |        |        |        |        |
| es | lo | 0.0379 | 0.9034 | 0.0586 |        |        |        |        |
| es | ml |        | 0.0257 | 0.9338 | 0.0404 |        |        |        |
| es | mi |        |        | 0.0217 | 0.9185 | 0.0598 |        |        |
| es | mh |        |        |        | 0.0505 | 0.8889 | 0.0606 |        |
| es | hi |        |        |        |        |        | 0.9027 | 0.0973 |
| es | hh |        |        |        |        |        | 0.0298 | 0.9702 |
| ms | ll | 0.9463 | 0.0488 | 0.0049 |        |        |        |        |
| ms | lo | 0.0336 | 0.8908 | 0.0756 |        |        |        |        |
| ms | ml |        | 0.0468 | 0.8763 | 0.0769 |        |        |        |
| ms | mi | 0.0045 |        | 0.0804 | 0.8661 | 0.0491 |        |        |
| ms | mh |        |        |        | 0.0735 | 0.8603 | 0.0588 | 0.0074 |
| ms | hi |        |        |        |        | 0.0504 | 0.8993 | 0.0504 |
| ms | hh |        |        |        |        |        | 0.0512 | 0.9488 |
| as | ll | 0.9428 | 0.0545 | 0.0027 |        |        |        |        |
| as | lo | 0.0408 | 0.8827 | 0.0765 |        |        |        |        |
| as | ml |        | 0.0373 | 0.8986 | 0.0642 |        |        |        |
| as | mi | 0.0027 |        | 0.0545 | 0.8883 | 0.0545 |        |        |
| as | mh |        |        |        | 0.0580 | 0.8750 | 0.0625 | 0.0045 |
| as | hi |        |        |        |        | 0.0278 | 0.9008 | 0.0714 |
| as | hh |        |        |        |        |        | 0.0387 | 0.9613 |

Appendix Table 3 Five States Markov Transition Matrix with Population Weight

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| wd | lo | 0.9058 | 0.0942 |        | 0.0000 |        |
| wd | ml | 0.0632 | 0.9125 | 0.0243 |        |        |
| wd | mi | 0.0022 | 0.0195 | 0.9441 | 0.0342 |        |
| wd | mh |        | 0.0000 | 0.0398 | 0.9395 | 0.0207 |
| wd | hi | 0.0000 |        | 0.0000 | 0.0024 | 0.9976 |
| ea | lo | 0.7134 | 0.2866 |        |        |        |
| ea | ml | 0.0636 | 0.9106 | 0.0258 |        |        |
| ea | mi |        | 0.0136 | 0.9653 | 0.0210 |        |
| ea | mh |        |        | 0.0330 | 0.9006 | 0.0664 |
| ea | hi |        |        |        |        | 1.0000 |
| eu | lo | 0.8857 | 0.1143 |        |        |        |
| eu | ml | 0.0175 | 0.8594 | 0.1232 |        |        |
| eu | mi |        | 0.0314 | 0.8570 | 0.1116 |        |
| eu | mh |        |        | 0.0251 | 0.9473 | 0.0276 |
| eu | hi |        |        |        | 0.0032 | 0.9968 |
| la | lo | 0.9567 | 0.0433 |        | 0.0001 |        |
| la | ml | 0.0074 | 0.9077 | 0.0849 |        |        |
| la | mi | 0.0000 | 0.0084 | 0.9592 | 0.0324 |        |
| la | mh |        | 0.0000 | 0.0456 | 0.9540 | 0.0005 |
| la | hi | 0.0003 |        | 0.0003 | 0.0490 | 0.9504 |
| me | lo | 0.9239 | 0.0761 |        |        |        |
| me | ml | 0.0306 | 0.8857 | 0.0837 |        |        |
| me | mi | 0.0035 | 0.0369 | 0.9226 | 0.0370 |        |
| me | mh |        |        | 0.0524 | 0.9358 | 0.0117 |
| me | hi |        |        |        | 0.0350 | 0.9650 |
| na | lo | 1.0000 |        |        |        |        |
| na | ml |        | 1.0000 |        |        |        |
| na | mi |        |        | 1.0000 |        |        |
| na | mh |        |        |        | 1.0000 |        |
| na | hi |        |        |        |        | 1.0000 |

Continues

|    |    | lo     | ml     | mi     | mh     | hi     |
|----|----|--------|--------|--------|--------|--------|
| sa | lo | 0.9403 | 0.0597 |        |        |        |
| sa | ml | 0.0781 | 0.9199 | 0.0020 |        |        |
| sa | mi |        | 0.0737 | 0.9255 | 0.0007 |        |
| sa | mh |        |        | 0.1903 | 0.8097 |        |
| sa | hi |        |        |        |        | 1.0000 |
| af | lo | 0.9807 | 0.0193 |        |        |        |
| af | ml | 0.0538 | 0.9268 | 0.0194 |        |        |
| af | mi | 0.0197 | 0.0446 | 0.8933 | 0.0423 |        |
| af | mh |        |        | 0.1109 | 0.8877 | 0.0014 |
| af | hi |        |        |        | 0.5000 | 0.5000 |
| nl | lo | 0.9567 | 0.0433 |        | 0.0001 |        |
| nl | ml | 0.0074 | 0.9077 | 0.0849 |        |        |
| nl | mi | 0.0000 | 0.0084 | 0.9592 | 0.0324 |        |
| nl | mh |        | 0.0000 | 0.0456 | 0.9540 | 0.0005 |
| nl | hi | 0.0000 |        | 0.0000 | 0.0002 | 0.9998 |
| es | lo | 0.8850 | 0.1150 |        |        |        |
| es | ml | 0.0675 | 0.9131 | 0.0194 |        |        |
| es | mi |        | 0.0155 | 0.9641 | 0.0204 |        |
| es | mh |        |        | 0.0331 | 0.9005 | 0.0663 |
| es | hi |        |        |        |        | 1.0000 |
| ms | lo | 0.9395 | 0.0605 |        |        |        |
| ms | ml | 0.0724 | 0.9158 | 0.0118 |        |        |
| ms | mi | 0.0031 | 0.0416 | 0.9230 | 0.0324 |        |
| ms | mh |        |        | 0.0526 | 0.9357 | 0.0117 |
| ms | hi |        |        |        | 0.0350 | 0.9650 |
| as | lo | 0.8864 | 0.1136 |        |        |        |
| as | ml | 0.0662 | 0.9121 | 0.0217 |        |        |
| as | mi | 0.0006 | 0.0192 | 0.9569 | 0.0233 |        |
| as | mh |        |        | 0.0409 | 0.9147 | 0.0444 |
| as | hi |        |        |        | 0.0045 | 0.9955 |

Appendix Table 4 Seven States Markov Transition Matrix with Population Weight

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| wd | ll | 0.9231 | 0.0769 | 0.0000 |        | 0.0000 |        |        |
| wd | lo | 0.0385 | 0.9153 | 0.0462 | 0.0000 |        |        |        |
| wd | ml | 0.0000 | 0.0484 | 0.9046 | 0.0470 |        | 0.0000 |        |
| wd | mi | 0.0032 | 0.0001 | 0.0213 | 0.9263 | 0.0492 | 0.0000 |        |
| wd | mh |        | 0.0000 | 0.0002 | 0.0481 | 0.9041 | 0.0475 | 0.0001 |
| wd | hi |        |        | 0.0000 | 0.0000 | 0.0403 | 0.9049 | 0.0548 |
| wd | hh | 0.0000 |        |        |        |        | 0.0164 | 0.9836 |
| ea | ll | 0.6988 | 0.3012 |        |        |        |        |        |
| ea | lo | 0.0323 | 0.9111 | 0.0566 |        |        |        |        |
| ea | ml |        | 0.0505 | 0.9066 | 0.0430 |        |        |        |
| ea | mi |        |        | 0.0001 | 0.9633 | 0.0366 |        |        |
| ea | mh |        |        |        | 0.0409 | 0.8747 | 0.0844 |        |
| ea | hi |        |        |        |        |        | 0.9188 | 0.0812 |
| ea | hh |        |        |        |        |        | 0.0460 | 0.9540 |
| eu | ll | 0.8434 | 0.1566 |        |        |        |        |        |
| eu | lo | 0.0467 | 0.7611 | 0.1923 |        |        |        |        |
| eu | ml |        | 0.0222 | 0.8903 | 0.0875 |        |        |        |
| eu | mi |        | 0.0013 | 0.0361 | 0.8259 | 0.1367 |        |        |
| eu | mh |        |        | 0.0005 | 0.0333 | 0.8954 | 0.0709 |        |
| eu | hi |        |        |        |        | 0.0353 | 0.9136 | 0.0511 |
| eu | hh |        |        |        |        |        | 0.0238 | 0.9762 |
| la | ll | 0.9713 | 0.0287 |        |        | 0.0001 |        |        |
| la | lo | 0.0215 | 0.9271 | 0.0513 | 0.0001 |        |        |        |
| la | ml | 0.0000 | 0.0086 | 0.8992 | 0.0922 |        | 0.0000 |        |
| la | mi |        |        | 0.0305 | 0.9293 | 0.0401 | 0.0000 |        |
| la | mh |        | 0.0000 |        | 0.0511 | 0.9229 | 0.0260 |        |
| la | hi |        |        | 0.0000 | 0.0000 | 0.2234 | 0.7711 | 0.0055 |
| la | hh | 0.0007 |        |        |        |        | 0.2332 | 0.7660 |
| me | ll | 0.9508 | 0.0490 | 0.0003 |        |        |        |        |
| me | lo | 0.0201 | 0.8324 | 0.1475 |        |        |        |        |
| me | ml |        | 0.0382 | 0.8907 | 0.0711 |        |        |        |
| me | mi | 0.0052 |        | 0.0455 | 0.8940 | 0.0553 |        |        |
| me | mh |        |        |        | 0.0573 | 0.9270 | 0.0149 | 0.0008 |
| me | hi |        |        |        |        | 0.0656 | 0.8849 | 0.0495 |
| me | hh |        |        |        |        |        | 0.0867 | 0.9133 |
| na | ll | 1.0000 |        |        |        |        |        |        |
| na | lo |        | 1.0000 |        |        |        |        |        |
| na | ml |        |        | 1.0000 |        |        |        |        |
| na | mi |        |        |        | 1.0000 |        |        |        |
| na | mh |        |        |        |        | 1.0000 |        |        |
| na | hi |        |        |        |        |        | 0.4722 | 0.5278 |
| na | hh |        |        |        |        |        | 0.0000 | 1.0000 |



Continues

|    |    | ll     | lo     | ml     | mi     | mh     | hi     | hh     |
|----|----|--------|--------|--------|--------|--------|--------|--------|
| sa | ll | 0.9370 | 0.0630 |        |        |        |        |        |
| sa | lo | 0.0448 | 0.9318 | 0.0234 |        |        |        |        |
| sa | ml |        | 0.0056 | 0.9900 | 0.0044 |        |        |        |
| sa | mi |        |        | 0.0567 | 0.9418 | 0.0015 |        |        |
| sa | mh |        |        |        | 0.1903 | 0.8097 |        |        |
| sa | hi |        |        |        |        |        | 1.0000 |        |
| sa | hh |        |        |        |        |        |        | 1.0000 |
| af | ll | 0.9815 | 0.0185 |        |        |        |        |        |
| af | lo | 0.0609 | 0.8733 | 0.0657 | 0.0001 |        |        |        |
| af | ml |        | 0.1614 | 0.8230 | 0.0156 |        |        |        |
| af | mi | 0.0223 |        | 0.0274 | 0.9025 | 0.0478 |        |        |
| af | mh |        |        |        | 0.1116 | 0.8871 | 0.0013 |        |
| af | hi |        |        |        |        | 0.1339 | 0.7991 | 0.0670 |
| af | hh |        |        |        |        |        | 0.6486 | 0.3514 |
| nl | ll | 0.9713 | 0.0287 |        |        | 0.0001 |        |        |
| nl | lo | 0.0215 | 0.9271 | 0.0513 | 0.0001 |        |        |        |
| nl | ml | 0.0000 | 0.0086 | 0.8992 | 0.0922 |        | 0.0000 |        |
| nl | mi |        |        | 0.0305 | 0.9293 | 0.0401 | 0.0000 |        |
| nl | mh |        | 0.0000 |        | 0.0511 | 0.9229 | 0.0260 |        |
| nl | hi |        |        | 0.0000 | 0.0000 | 0.2234 | 0.7710 | 0.0056 |
| nl | hh | 0.0000 |        |        |        |        | 0.0003 | 0.9997 |
| es | ll | 0.8957 | 0.1043 |        |        |        |        |        |
| es | lo | 0.0375 | 0.9198 | 0.0427 |        |        |        |        |
| es | ml |        | 0.0459 | 0.9150 | 0.0391 |        |        |        |
| es | mi |        |        | 0.0016 | 0.9627 | 0.0357 |        |        |
| es | mh |        |        |        | 0.0411 | 0.8746 | 0.0843 |        |
| es | hi |        |        |        |        |        | 0.9188 | 0.0812 |
| es | hh |        |        |        |        |        | 0.0460 | 0.9540 |
| ms | ll | 0.9379 | 0.0621 | 0.0000 |        |        |        |        |
| ms | lo | 0.0438 | 0.9279 | 0.0283 |        |        |        |        |
| ms | ml |        | 0.0221 | 0.9398 | 0.0381 |        |        |        |
| ms | mi | 0.0047 |        | 0.0466 | 0.8986 | 0.0501 |        |        |
| ms | mh |        |        |        | 0.0575 | 0.9268 | 0.0149 | 0.0008 |
| ms | hi |        |        |        |        | 0.0656 | 0.8849 | 0.0495 |
| ms | hh |        |        |        |        |        | 0.0867 | 0.9133 |
| as | ll | 0.8988 | 0.1012 | 0.0000 |        |        |        |        |
| as | lo | 0.0372 | 0.9183 | 0.0444 |        |        |        |        |
| as | ml |        | 0.0452 | 0.9128 | 0.0421 |        |        |        |
| as | mi | 0.0010 |        | 0.0103 | 0.9491 | 0.0396 |        |        |
| as | mh |        |        |        | 0.0481 | 0.8973 | 0.0543 | 0.0004 |
| as | hi |        |        |        |        | 0.0107 | 0.9132 | 0.0761 |
| as | hh |        |        |        |        |        | 0.0507 | 0.9493 |