# Do Non-Prudent Consumers Ever Engage in Precautionary Saving? Two Observations on Risk and Precautionary Saving

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**Abstract**

In this paper, we first show that a particular form of precautionary saving, which we will call "intertemporal precautionary saving" to distinguish it from purely intertemporal and purely precautionary saving, will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, even if they are not prudent. We then present a simple example that shows that even pure precautionary saving (i.e., saving generated by risk alone without effects on expected income) may arise as long as consumers are risk-averse, even if they are not prudent and even if risk is speculative (two-sided).

*Journal of Economic Literature* classification codes: D11, D14, D15, D81, E21, G51

Keywords: household saving, precautionary saving, prudence, pure risk, risk aversion, saving, speculative risk

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## 1. Introduction

Precautionary saving arising from income risk and various other uncertainties is an important component of household saving. Countless empirical papers have been written about the importance of precautionary saving arising from future income risk, and as the excellent survey by Lugilde, et al. (2019, p. 507) shows, most of these studies find "robust and convincing results as regards the existence of a precautionary motive for saving" although "there is not a consensus on the magnitude of this effect." For example, Skinner (1988) finds, using data from the Panel Study of Income and Dynamics, that 56% of total household saving in the United States is attributable to precautionary saving arising from income risk, while Dardanoni (1991) finds, using data from the U.K. Family Expenditure Survey, that the comparable figure for the United Kingdom is more than 60% (see Zhou, 2003, for a similar study for Japan). Similarly, Carroll and Samwick (1998) find, using data from the Panel Study of Income Dynamics, that between 32-50% of wealth in their sample of American households is attributable to the extra uncertainty that some consumers face compared to the lowest uncertainty group. A different approach is taken by Horioka and Watanabe (1997), Horioka, et al. (2000), and Horioka and Ventura (2025), who use data from household surveys to analyze the relative importance of various motives for household saving. They find that a substantial proportion of households are saving for precautionary purposes in Japan, Europe, and the United States and furthermore that the quantitative importance of such saving is often quite substantial. Moreover, saving for income risk is presumably an important component of precautionary saving, which also includes saving for illness, accidents, longevity risk, and other unforeseen contingencies.

Thus, it is important to have a profound understanding of the theory of precautionary saving in general and of the impact of income risk on household saving in particular. There have been many theoretical analyses of precautionary saving starting with the seminal paper by Leland (Leland, 1968) more than half a century ago (see Jappelli and Pistaferri (2017, Chapter 6) and Baiardi, et al., 2020, for surveys of this literature). Leland (1968) was the first to identify the conditions required for income variance to stimulate saving. In particular, he showed that risk aversion alone is not enough for saving to respond to increasing fluctuations of income around an expected future value and that an additional condition is required beyond risk aversion—namely, the convexity of marginal utility or $U'''(.) > 0$ (a positive third derivative of the utility function or "positive prudence," to use Kimball's (1990) terminology) (see Sandmo, 1970, for a closely related analysis).

Kimball (1990) extended Leland's (1968) analysis by defining the coefficient of absolute prudence as minus the third derivative of the utility function divided by its second derivative and conducting an analysis akin to the one developed for risk aversion by Arrow and Pratt, who defined the coefficient of absolute risk aversion as minus the second derivative of the utility function divided by its first derivative (see, for example, Pratt, 1964). In particular, just as Arrow and Pratt showed that the coefficient of absolute risk aversion is a good measure to gauge the extent of risk aversion, Kimball (1990) showed that the coefficient of absolute prudence is a good measure to quantify the intensity of precautionary behavior (or, to use Kimball's own words, "the propensity to prepare and forearm oneself in the fact of uncertainty about future income"). He did so by basically transposing Arrow and Pratt's analysis to the new context (by replacing total utility with marginal utility) and defining new concepts such as the coefficients of absolute and relative prudence (corresponding to the coefficients of absolute and relative risk aversion, the "precautionary premium" (corresponding to the "risk premium"), etc.

By contrast, Yaari (1987) proved that, within the anticipated utility framework formulated by Quiggin (1982), a risk-averse consumer will engage in precautionary saving regardless of the sign of the third derivative of its utility function (i.e., regardless of whether or not it is prudent). Moreover, Segal et al. (1988) extended a modification of this result to an n-period model.

In this paper, we conduct a theoretical analysis of precautionary saving and show that, even in the more standard context of expected utility, there are situations in which precautionary saving will arise as long as consumers are risk-averse, even if they are not prudent.

Previous researchers on precautionary saving have employed a "speculative risk" concept of income whereby there is downside risk as well as upside risk (i.e., there is the possibility of income gains as well as income losses). In this paper, we focus on a "pure risk" concept of income whereby there is only downside risk (i.e., there is only the possibility of a loss of income) (see, for example, Willett, 1901, and Knight, 1921). Note that this type of income risk is quite common in actual practice. For example, in the case of the risk of unemployment, there is the possibility of a loss of income but not the possibility of an increase in income. The same is true of health problems, accidents involving property and vehicles, etc.

The contribution of this paper is twofold. First, we show that a particular form of precautionary saving, which we will call "intertemporal precautionary saving" to distinguish it from purely intertemporal and purely precautionary saving, will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, even if they are not prudent. However, we also show that prudence *will* affect the intensive margin (the *amount* of intertemporal precautionary saving that consumers do) even though it does not affect the extensive margin (whether or not consumers engage in intertemporal precautionary saving).

Whereas our first contribution is to show that a weaker form of precautionary saving will arise as long as consumers are risk-averse, even if they are not prudent, in the case of pure (downside) risk, our second contribution is to present a simple example that shows that even pure precautionary saving (i.e., saving generated by risk alone without effects on expected income) may arise as long as consumers are risk-averse, even if they are not prudent and even if risk is speculative (two-sided). To do so, we use a more general Arrow-Debreu economy in which saving is fully integrated in the set of assets available to individuals to transfer income and consumption across date-event pairs and helps consumers to achieve perfect insurance. As above, prudence *will* affect the intensive margin (the *amount* of precautionary saving that consumers do) even though it does not affect the extensive margin (whether or not consumers engage in precautionary saving).

## 2. A Model of Intertemporal Precautionary Saving

Consider an intertemporal economy lasting for two periods, 0 and 1, with consumption in period 0 and consumption in period 1 in $c = 1, \ldots, C$ possible contingencies (states of the world). In period 0, consumers can save an amount $s$ out of their endowment. For simplicity, initial endowment is $w$ in period 0 and in period 1 for all contingencies. For simplicity, both the time preference factor and the interest factor (1 + interest rate) are set equal to one so that there is no incentive to save for intertemporal motives.

Without loss of generality, pure risk is represented by a loss $\epsilon$ in one state of the world, $c^{\prime}$, with probability $\pi_{c^{\prime}}$. The instantaneous utility function is $u(x)$, continuously twice differentiable and strictly concave, so that consumers are risk-averse.

Proposition 1. In the presence of pure risk, a risk-averse consumer engages in intertemporal precautionary saving even if she is not prudent.

Proof. The intertemporal maximization problem of a consumer is:

$max_s\, u(w-s) + \sum_{c\neq c^{\prime}} \pi_c u(w+s) + \pi_{c^{\prime}}\, u(w-\epsilon+s)$

The first-order condition with respect to $s$ is:

$F(w,\epsilon,s) = -u'(w-s) + \sum_{c\neq c'} \pi_c u'(w+s) + \pi_{c'} u'(w-\epsilon+s) = 0$ (1)

(The second-order condition holds because of the concavity of $u(x)$.)

It is clear that (1) is solved by $s = 0$ when $\epsilon = 0$ as (1) becomes

$F(w,0,0) = -u'(w) + \sum_{c\neq c'} \pi_c u'(w) + \pi_{c'} u'(w) = -u'(w) + \sum_c \pi_c u'(w) = -u'(w) + u'(w) = 0$.

By concavity of $u(x)$, when $\epsilon > 0$ $u^{\prime}(w-\epsilon)$ is greater than $u^{\prime}(w)$ (how much greater depends on the concavity of $u(x)$) and therefore $F(w,\epsilon,0) > 0$.

By concavity of $u(x)$, we have that $\frac{\partial F(w,\epsilon,s)}{\partial s} = u''(w-s) + \sum_{c\neq c^{\prime}} \pi_c u''(w+s) + \pi_{c'} u''(w-\epsilon+s) < 0$ since all of the terms in the summation are negative.

Hence, there will exist a value of $s^* > 0$ such that the first-order condition is satisfied--i.e.,

$F(w,\epsilon,s^*) = 0$.

Thus, Proposition 1 shows that, facing pure risk, the consumer will engage in intertemporal precautionary saving, i.e., saving induced by a fall in expected income, regardless of whether or not she is prudent. Pure risk has been represented by a shock in just one state of the world, but the proof can easily be generalized to the case in which the shock occurs in multiple states.

The intuition underlying this result is quite straightforward. Consumers allocate consumption across the two periods by equating marginal utility of consumption across periods. Since period 1 features many possible states of the world, marginal utility in period 0 will be equated to average marginal utility in period 1. A fall in the endowment of at least one state of the world in period 1will increase marginal utility of consumption in that state and consequently increase average marginal utility. To restore equality, consumption in period 0 must decrease (i.e., saving must increase) and consumption in period 1 must increase, on average.

Lastly, expression (1) also shows that $s < \epsilon$ as, if $s = \epsilon$, we would obtain:

$F(w,\epsilon,\epsilon) = -u'(w-\epsilon) + \sum_{c\neq c'} \pi_c u'(w+\epsilon) + \pi_{c'} u'(w) < 0$[1]

Thus, $s$ would have to decrease to restore equality in light of Proposition 1.

However, prudence will contribute toward determining the intensity of intertemporal precautionary saving, as shown by the following:

Proposition 2. The derivative of intertemporal precautionary saving with respect to $\epsilon, \frac{ds(\epsilon)}{d\epsilon}$, depends positively on absolute prudence.

Proof. By the implicit function theorem and the continuous differentiability of $F(w,\epsilon,s)$, we know that there exists a continuous function $s(\epsilon)$ in a neighborhood of each solution of the utility maximization problem and that its derivative exists and is given by:

$$\frac{ds}{d\epsilon} = -\frac{\frac{\partial F}{\partial \epsilon}}{\frac{\partial F}{\partial s}} .$$

$\frac{\partial F}{\partial \epsilon} = -\pi_{c'} u''(w-\epsilon+s) \cong -\pi_{c'}(u''(w) - u'''(w)(\epsilon - s))$ by a first-order Taylor expansion around $w$.

Also,

$$\begin{aligned}
\frac{\partial F}{\partial s} &= u''(w-s) + \sum_{c\neq c'} \pi_c u''(w+s) + \pi_{c'} u''(w-\epsilon+s) \\
&\cong u''(w) - u'''(w)s \\
&\quad + \sum_{c\neq c'} \pi_c\big(u''(w) + u'''(w)s\big) + \pi_{c'}\big(u''(w) + u'''(w)(s-\epsilon)\big) \\
&= u''(w) - u'''(w)s \\
&\quad + \sum_{c\neq c'} \pi_c u''(w) + \sum_{c\neq c'} \pi_c u'''(w)s + \pi_{c'} u''(w) + \pi_c u'''(w)s - \pi_c u'''(w)\epsilon \\
&= 2u''(w) - \pi_{c'} u'''(w)\epsilon
\end{aligned}$$

by using first-order Taylor expansions around $w$ and performing some algebraic manipulations.

Their ratio yields:

$$\frac{ds}{d\epsilon} = -\frac{\frac{\partial F}{\partial \epsilon}}{\frac{\partial F}{\partial s}} \cong -\frac{-\pi_{c'}(u''(w) - u'''(w)(\epsilon - s))}{2u''(w) - \pi_{c'} u'''(w)\epsilon} = \frac{\pi_{c'}(u''(w) - u'''(w)(\epsilon - s))}{2u''(w) - \pi_{c'} u'''(w)\epsilon} .$$

Dividing both the numerator and the denominator by $u''(w)$ yields:

$$\frac{ds}{d\epsilon} \cong \frac{\pi_{c'}(1+\rho(w)(\epsilon - s))}{2+\pi_{c'}\rho(w)\epsilon} ,$$

where $\rho(w)$ is Kimball's (1990) coefficient of absolute prudence evaluated at w.

One can easily show that $\frac{ds}{d\epsilon}$ is increasing in absolute prudence as

[1] To see this, just consider that in view of concavity of $u(w)$, $\sum_{c\neq c'} \pi_c u'(w+\epsilon) + \pi_{c'} u'(w) < u'(w) < u'(w-\epsilon)$.

$\frac{d}{d\rho}\left(\frac{ds}{d\epsilon}\right) = \frac{2\pi_{c'}(\epsilon - s) + \pi_{c'}^2 \epsilon}{(2+\pi_{c'}\rho(w)\epsilon)^2} > 0$ , as $(\epsilon - s) > 0$ in light of the observation at the end of Proposition1.

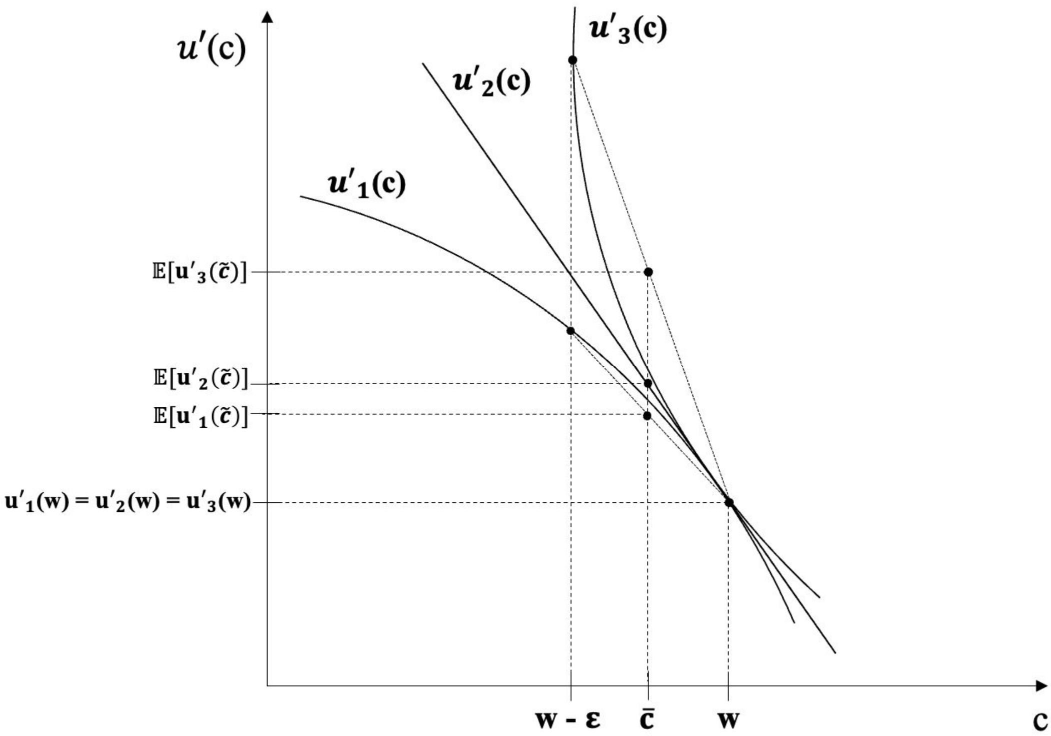
The intuition underlying Proposition 2 is also straightforward. Prudence affects the sensitivity of saving to the shock because the fact that marginal utility is convex (positive prudence) or concave (negative prudence) implies that the marginal utility of consumption in the state affected by the shock will increase more quickly (convex marginal utility, positive prudence) or more slowly (concave marginal utility) than with linear marginal utility (zero prudence).

Figure 1 summarizes the intuition of both propositions. The solid curves represent the marginal utilities of three different utility functions, characterized by varying levels of prudence. At consumption w, which is the endowment of both period 0 and state 1 in period 1, marginal utility is the same for the three functions. As the three curves are tangent at w, the second derivatives are also the same. Thus, absolute risk aversion is equal across utility functions at w. Prudence is not, as the first marginal utility is concave, the second is linear, and the third is convex (corresponding to negative, zero and positive absolute prudence).

Figure 1 goes here

The main intuition of Proposition 1 is illustrated by the fact that average marginal utility for the three curves is always higher than marginal utility at w in the face of an expected loss. Therefore, consumption is optimally shifted from period 0, when marginal utility is $u'(w)$, to period 1, when expected marginal utility is higher.

**Figure 1. Risk Aversion, Prudence, and Precautionary Saving**



How much higher depends on the curvature of $u'(w)$, which is the main content of Proposition 2. Expected marginal utility is lowest for the concave marginal utility function and highest for the convex one.

## 3. Saving without Prudence with Pure Risk: An Example

Let us consider an economy with consumption in period 0 and in period 1 with two equally probable states in period 1, called 1 and 2.

Preferences are represented by an intertemporally separable utility function, where instantaneous utility is quadratic and equal to $u(x) = 100x - x^2$. The third derivative of the utility function is zero, so our consumer does not display prudence, only risk aversion.

Endowments are equal to 20 in all periods and states.

Pure risk materializes as a loss in state 2, represented by a coefficient $k \in [0,1]$ that multiplies the endowment in that state, i.e., w = 20k. If k = 1, there is no loss in state 2; if k = 0, the loss in state 2 is total (zero endowment).

The maximization problem is thus:

$$\max 100x_0 - x_0^2 + \frac{1}{2}(100x_1 - x_1^2) + \frac{1}{2}(100x_2 - x_2^2)$$

such that

$$x_0 = 20 - s$$

$$x_1 = 20 + s$$

$$x_2 = 20k + s$$

Substituting the constraints into the objective function, we obtain the equivalent problem:

$$max_s\, 100(20 - s) - (20 - s)^2 + \frac{1}{2}(100(20 + s) - (20 + s)^2) + \frac{1}{2}(100(20k + s) - (20k + s)^2)$$

with the following first-order conditions:

$$-100 + 2(20 - s) + \frac{1}{2}\big(100 - 2(20 + s)\big) + \frac{1}{2}\big(100 - 2(20k + s)\big) = 0 \text{ , i.e.}$$

$$-100 + 40 - 2s + \frac{1}{2}(100 - 40 - 2s + 100 - 40k - 2s) = 0 \text{ , i.e.}$$

$20 - 4s - 20k = 0$ , i.e. $20(1 - k) = 4s$, yielding $s = 5(1 - k)$.

This implies that, in the absence of a loss (i.e., k = 1), $s = 0$, whereas when the loss is complete (i.e., k = 0), $s = 5$.

Why does risk aversion alone generate precautionary saving in the case of pure risk?

Pure risk decreases consumption in one state of the world, leaving consumption in the other state unaffected and equal to consumption in period 0. This means that the average marginal utility of income in period 1 will certainly be higher than marginal utility in period 0. This will make our consumer willing to shift consumption from period 0 to period 1--i.e., to engage in this weaker form of precautionary saving.

Thus, in the case of pure risk, prudence (the third derivative of the utility function), which describes the rate at which marginal utility increases in the bad state, will affect the amount of saving that is induced by risk aversion. In other words, in the case of pure risk, intertemporal precautionary saving will arise as long as consumers are risk-averse, regardless of whether or not they are prudent. This also means, of course, that with pure risk, we may have positive saving in the presence of downside risk even if prudence is negative.

## 4. Precautionary Saving in a Simple Arrow-Debreu Economy without Prudence: An Example

Whereas our observation in the previous section can be regarded as being a simple observation about saving and risk that matters only because pure (downside) risk is extremely important in actual practice, the example we present in this section makes the theoretically more important point that if saving is considered, as it should, as part of the financial structure of an economy, (pure) precautionary saving may arise even if consumers are not prudent and even if risk is speculative (two-sided) as long as consumers are risk-averse.

In this section, we present a simple example, cast in a stylized Arrow-Debreu economy, in which saving reacts positively to speculative (two-sided) risk with zero mean. We consider an exchange economy with two agents, in which one agent has linear preferences, so that she is risk-neutral, while the preferences of the other agent are concave and quadratic. Hence, this agent is risk-averse and displays no prudence (the third derivative of her utility function is equal to zero).

The economy features one consumption good in period 0, and one consumption good in each of two states of the world in period 1. Agents can transfer consumption across date-event pairs by trading two assets in "zero net supply" (defined as assets for which the sum across agents of portfolio holdings of these assets is zero) that pay off in period 1. One is an Arrow-Debreu asset, paying one unit of consumption in state 1 only, while the other is a riskless asset, paying one unit of consumption in both states in period 1. Buying this asset is thus equivalent to saving, while selling this asset is equivalent to lending. For simplicity, the time discount factor is set equal to unity, and probabilities are equal. The prices of the two assets are $q_1$ and $q_2$, respectively.

The preferences of the risk-averse agent are represented by the intertemporal expected utility function:

$$U(x_0, x_1, x_2) = 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right).$$

This agent has an initial endowment of 4 in the first period, and state contingent endowments of $4 + \epsilon$ and $4 - \epsilon$, respectively, in the two states in period 1. This agent's income is therefore affected by a speculative, symmetric risk with zero mean and variance equal to $\varepsilon^2$.

The maximization problem of this agent is:

$max_{x_0, x_1, x_2}\, 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right)$

such that:

$$x_0 = 4 - q_1 y_1 - q_2 y_2$$

$$x_1 = 4 + \epsilon + y_1 + y_2$$

$$x_2 = 4 - \epsilon + y_2 \;.$$

By substituting the constraints into the utility function, we obtain the equivalent problem:

$max_{y_1, y_2}\, 5(4 - q_1 y_1 - q_2 y_2) - \frac{1}{2}(4 - q_1 y_1 - q_2 y_2)^2 \; + \frac{1}{2}\left(5(4 + \epsilon + y_1 + y_2) - \frac{1}{2}(4 + \epsilon + y_1 + y_2)^2 \right) + \frac{1}{2}\left(5(4 - \epsilon + y_2) - \frac{1}{2}(4 - \epsilon + y_2)^2 \right)$

with first-order conditions with respect to asset holdings $(y_1, y_2)$:

$$-5q_1 + (4 - q_1 y_1 - q_2 y_2)q_1 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - y_2) = 0$$

$$-5q_2 + (4 - q_1 y_1 - q_2 y_2)q_2 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - y_2) + \frac{1}{2}(5 - 4 + \epsilon - y_2) = 0.$$

By solving the problem of the risk-neutral agent, we immediately obtain that the equilibrium asset prices are $q_1 = \frac{1}{2}$ and $q_2 = 1$. Thus, the presence of the risk-neutral agent allows a simple closure of the model.

By substituting equilibrium asset prices into the first-order conditions and rearranging, we obtain:

$$-\frac{3}{4} y_1 - y_2 - \frac{\epsilon}{2} = 0$$

$$y_2 = -\frac{1}{2} y_1.$$

Solving this system of equations yields:

$$y_1 = -2\varepsilon \;,\; y_2 = \epsilon.$$

We immediately notice that the demand for saving, i.e. for the riskless asset, is a linear function of $\epsilon$.

Saving is therefore a function of the (symmetric) shock, and it can be fully characterized as purely precautionary, as the income shock has zero mean.

Lastly, by substituting asset holdings in the budget constraint, we obtain a final allocation for this agent of (4,4) in the second period, i.e., the risk-neutral agent achieves perfect insurance.

The reason why this is so is that, in this stylized Arrow-Debreu economy, saving is carried out by buying a risk-free asset, which crucially contributes to expanding the insurance opportunities available to the agents and, consequently, the opportunities to transfer consumption across dates and contingencies. In this example, markets are complete, and they are so thanks to the presence of the riskless asset (saving) in addition to a financial asset. Risk aversion, in and by itself, may thus well generate precautionary saving, even in the absence of prudence. The only situation in which the effect on saving highlighted in our example would not arise is if markets were complete to begin with, even in the absence of a riskless asset, a circumstance that we can safely rule out in actual practice.

Three clarifying remarks are in order concerning the result shown in the example.

Remark 1. The presence of an Arrow-Debreu asset, instead of a generic risky asset, is immaterial, in the sense that similar results may be obtained by using a more general asset with state contingent returns.

Remark 2. The fact the saving is affected via a traded asset (credit markets) instead of a simpler storage technology (self saving) is also immaterial. In the Appendix we present the solution to the problem where saving is represented by a simple storage technology, and the solution is identical to the one obtained above.

Remark 3. Assets are in zero net supply, i.e., they only constitute a means to shift income and consumption across dates and states; therefore, they do not represent additional risks. The only risk present in our economy is the endowment risk $(+\epsilon, -\epsilon)$.

## 5. Concluding Remarks

In this paper, we first conducted a theoretical analysis of precautionary saving in the case of pure (downside) risk and showed that a weaker form of precautionary saving, which we named "intertemporal precautionary saving" to distinguish it from purely intertemporal and purely precautionary saving, will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, even if they are not prudent. We also showed that prudence *will* affect the intensive margin (the *amount* of intertemporal precautionary saving that consumers do) even though it does not affect the extensive margin (whether or not consumers engage in precautionary saving).

Although these findings may be considered as a simple observation on saving and risk, we note that pure (downside) risk more often motivates consumers' choices (see, for example, the unemployment, health, and accident examples in the introduction) and that speculative risk (including both downside and upside risk) is more relevant only for financial investors. Thus, the findings of this paper have wide applicability (including in the above examples) and imply that any measure that mitigates pure (downside) risk will always bring about a reduction in precautionary saving.

A more important finding of the paper from a theoretical viewpoint is that even pure precautionary saving (i.e., saving generated by risk alone without effects on expected income) may arise as long as consumers are risk-averse, even if they are not prudent and even if risk is speculative (two-sided), using a more general Arrow-Debreu economy in which saving is fully integrated in the set of assets available to individuals to transfer income and consumption across date-event pairs and helps consumers to achieve perfect insurance. As above, prudence *will* affect the intensive margin (the *amount* of precautionary saving that consumers do) even though it does not affect the extensive margin (whether or not consumers engage in precautionary saving).

From a more empirical standpoint, the findings of this paper imply that measures of risk aversion (which are much more readily available than measures of prudence) may constitute an extremely relevant control variable in any saving equation.

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## Appendix: A Model of Saving with a Storage Technology

Let us restate the model of section 4 without a riskless asset (asset 2 in the example), which allows saving to materialize with a storage technology instead.

s is the amount of saving realized by the risk averse agent, i.e., the amount of consumption forgone in period 0 and transferred to period 1. Again, for simplicity, the time preference and interest factors are set equal to one.

Then, the maximization problem of the risk averse agent becomes:

$$max_{x_0, x_1, x_2}\, 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right)$$

such that:

$$x_0 = 4 - q_1 y_1 - s$$

$$x_1 = 4 + \epsilon + y_1 + s$$

$$x_2 = 4 - \epsilon + s\ .$$

By substituting these constraints into the utility function, we obtain the equivalent problem:

$$max_{y_1, y_2}\, 5(4 - q_1 y_1 - s) - \frac{1}{2}(4 - q_1 y_1 - s)^2 + \frac{1}{2}\left(5(4 + \epsilon + y_1 + s) - \frac{1}{2}(4 + \epsilon + y_1 + s)^2\right) + \frac{1}{2}\left(5(4 - \epsilon + s) - \frac{1}{2}(4 - \epsilon + s)^2\right)$$

with first-order conditions with respect to asset holdings $(y_1, y_2)$:

$$-5q_1 + (4 - q_1 y_1 - s)q_1 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - s) = 0$$

$$-5q_2 + (4 - q_1 y_1 - s) + \frac{1}{2}(5 - 4 - \epsilon - y_1 - s) + \frac{1}{2}(5 - 4 + \epsilon - s) = 0.$$

By solving the problem of the risk-neutral agent, we can immediately show that the equilibrium price of the risky asset is $q_2 = 1/2$. Thus, the presence of the risk-neutral agent allows a simple closure of the model.

By substituting equilibrium asset prices into the first-order conditions and rearranging, we obtain:

$$-\frac{3}{4}y_1 - s - \frac{\epsilon}{2} = 0$$

$$s = -\frac{1}{2}y_1.$$

Solving this system of equations yields:

$$y_1 = -2\varepsilon\ , s = \epsilon.$$

We immediately notice that saving is a linear function of $\epsilon$. This is bona fide precautionary saving, as it is generated by speculative, zero-mean risk in the absence of prudence (zero third derivative).